MachineLearnAthon - Microlecture
Regression analysis quide

MachineLearnAthon
A project Co-funded by the Erasmus+ programme of the European Union

Steps of regression analysis:

1. **Choose a model:**

Select a model that best describes the relationship between inputs (independent variables) and the output (dependent variable).

- **Linear Regression**
  Assumes a straight-line relationship: An increase in the input leads to an increase or decrease in the output in a constant proportion.

- **Non-Linear Regression**
  Captures relationships where the effect of input on output changes depending on the input level.Used when patterns cannot be adequately described by a straight line.

Choose a model

Estimate regression function

"Check it out." regression function

Steps of regression analysis:

2. **Estimate the regression function:**

- There is no single statistical test definitively says “use linear” or “use polynomial” regression.
- There are diagnostic tools and model comparison tests that help you decide whether a linear model is sufficient or if a polynomial (nonlinear) regression fits the data better.

Choose a model

Estimate
regression function

"Check it out."
regression function

Steps of regression analysis:

3. **Check the quality of the estimate:**

   - How well does the found model fit to explain the "response" (target variable)?
   - What contribution do the different factors of influence have in explaining the response?

   Look for:

   - R-squared
   - P-values
   - AIC/BIC

Choose a model

Estimate regression function

"Check it out." regression function

Example: margarine shop

- The sales manager of a large margarine manufacturer wonders whether the turnover achieved is due to the level of advertising costs!?

- The source of the data used are: Backhaus/Erichson/Plinke/Weiber - Multivariate Analysemethoden, Springer Verlag 2000

Quantity per year	Price per carton	Advertising per year	Customer visits per year
2 585	12,50	2 000	109
1 819	12,00	550	107
1 647	9,95	1 000	99
1 496	11,50	800	70
921	12,00	0	81
2 278	10,00	1 500	102
1 810	8,00	800	110
1 987	9,00	1 200	92
1 612	9,50	1 100	87
1 913	12,50	1 300	79

Example: margarine shop

Choose a model

$$\hat{Y} = b_0 + b_1 \times X$$

Estimate regression function

$$Y_i = b_0 + b_1 - X_i + e_i$$

The observed value of Y_i consists of:

- The proportion of $b_0 + b_1 - X_i$ quantity sold that can be explained by the regression model.
- Residual variation in the quantity sold e_i, which cannot be explained by the model (Residuum).

"Check it out." regression function

R-squared: 0.829					Adj. R-squared: 0.808			
	x_i	b_i	b_i [low]	b_i [high]	s_i	|t|	P
Constant	f(x1, x2)	1036.373	717.339	1355.407	138.349	7.491	0.0
Advertising per year	X1	0.752	0.474	1.030	0.121	6.235	0.0

b_0 b_1

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Example: margarine shop

Choose a model

Estimate
regression function

"Check it out."
regression function

	B = R-squared: 0.829				Adj. R-squared: 0.808		
	x_i	b_i	b_i [low]	b_i [high]	s_i	|t|	P
Constant	f(x1, x2)	1036.373	717.339	1355.407	138.349	7.491	0.0
Advertising per year	X1	0.752	0.474	1.030	0.121	6.235	0.0

B ("certainty factor") = R-Sq (= R²)

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(Y-\hat{Y}\right)^2}{n-2}}$$

$$R^2 = \frac{\sum_{i=1}^{n}\left(\hat{Y}-\bar{Y}\right)^2}{\sum_{i=1}^{n}(Y-\bar{Y})^2}$$

Dummy Variable Added

- **Reason:** A "dummy" variable (binary variable) was added to capture whether the advertising budget is above or below the median value.
- **Definition:**
  - HighAdvertising = 1 → Advertising budget is above the median.
  - HighAdvertising = 0 → Advertising budget is below or equal to the median.
- **Purpose:** The dummy variable was included to account for possible non-linear effects in the relationship between advertising budget and sales.

Assessment of the model

Basic Model Statistics

	Metric	Value
0	Coefficient of Determination (R)	93.28%
1	Adjusted Coefficient of Determination (R*)	87.90%
2	Residual Variance (s^2)	24418.68
3	Residual Standard Deviation (s)	156.26

Test for Independence of Predictor (F-test)

	Significance Level α	Critical Value	Test Statistic	Result
0	5.0%	5.190000	17.340000	Reject H0
1	1.0%	11.390000	17.340000	Reject H0
2	0.1%	31.090000	17.340000	Do not reject H0

Ramsey RESET Test

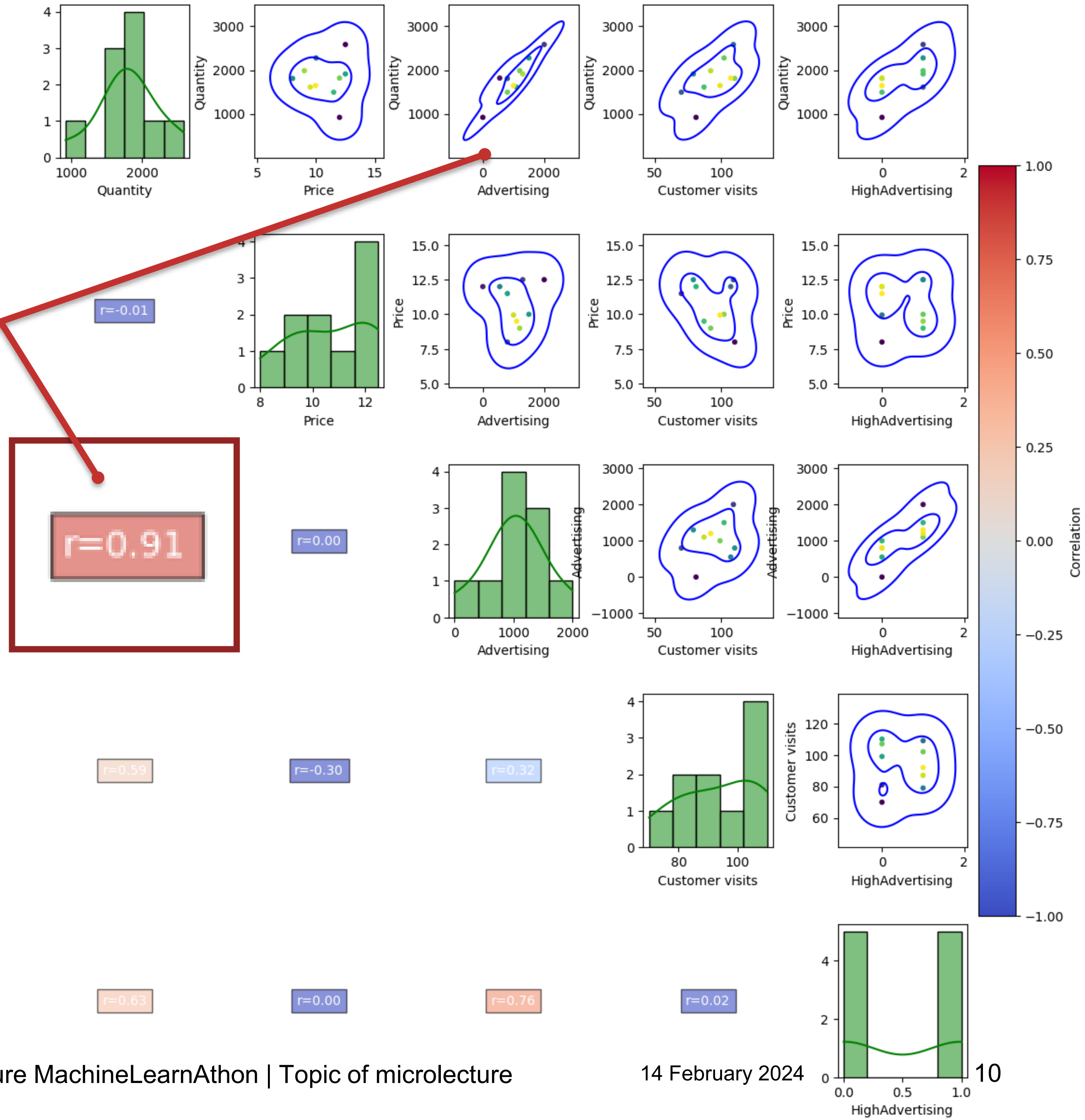
	Significance Level α	Critical Value	Test Statistic	Result
0	5.0%	5.190000	0.760000	Do not reject H0
1	1.0%	11.390000	0.760000	Do not reject H0
2	0.1%	31.090000	0.760000	Do not reject H0

Conclusions

	Test	Test Statistic	p-value	Conclusion
0	Test for Independence of Predictor	17.340000	0.003900	We reject the null hypothesis at least at one of the tested significance levels.
1	Ramsey RESET Test	0.760000	0.382520	We cannot reject the null hypothesis at any of the tested significance levels.

Example: margarine shop

There could be a dependency



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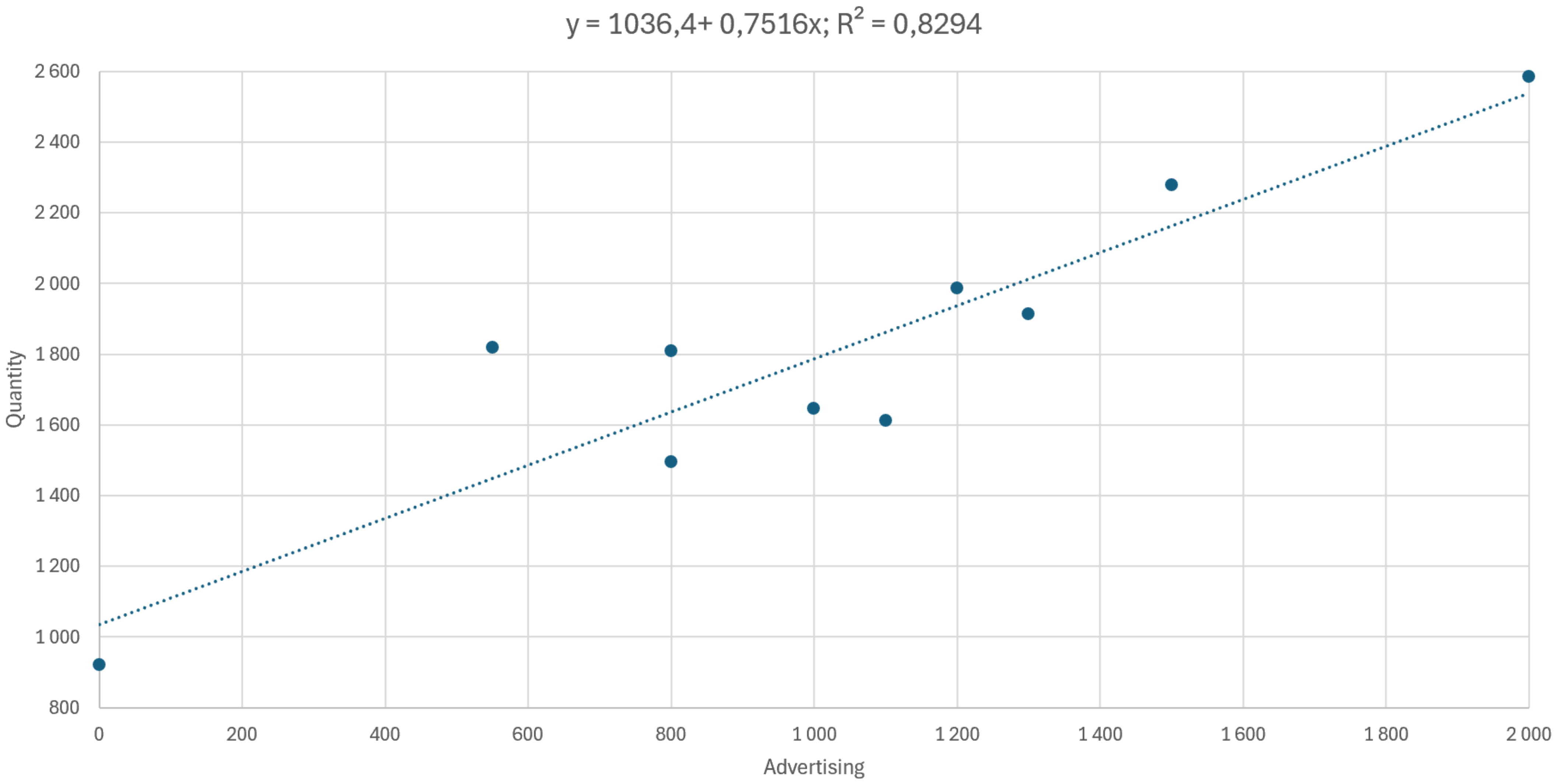
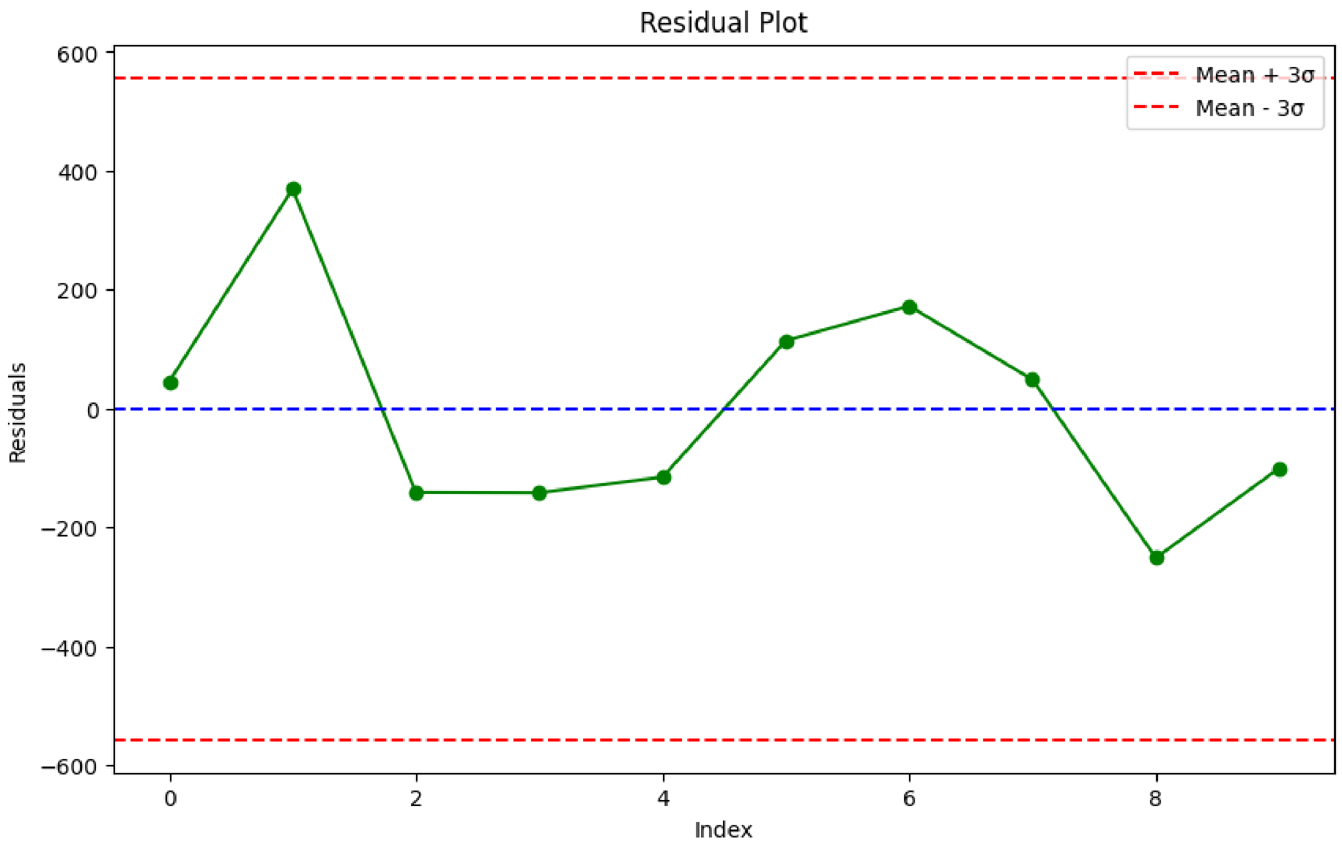
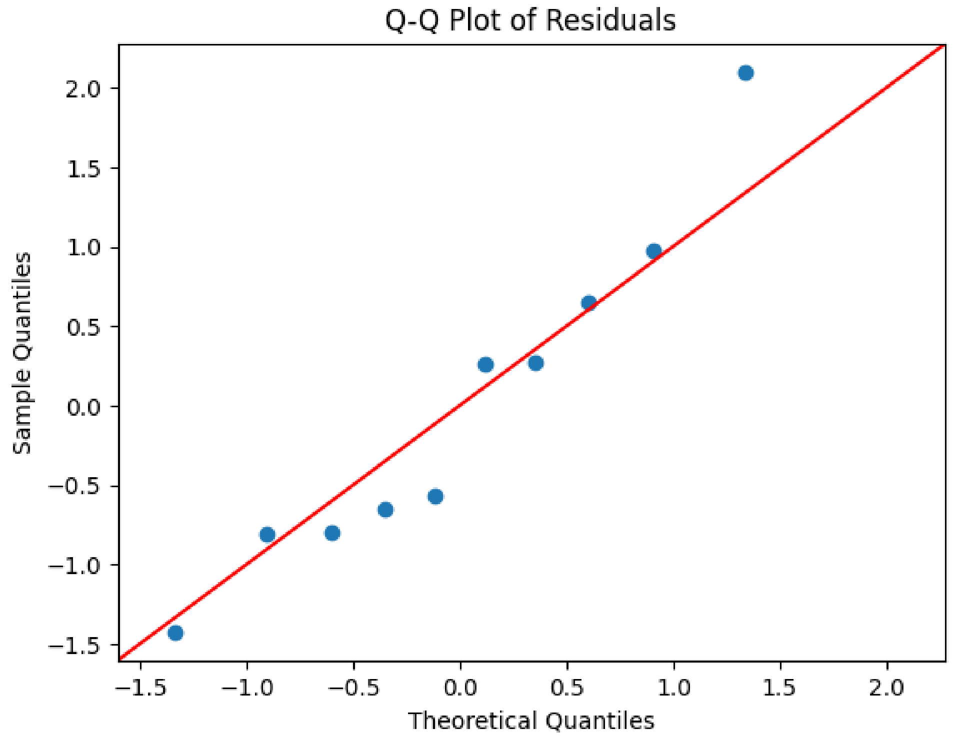
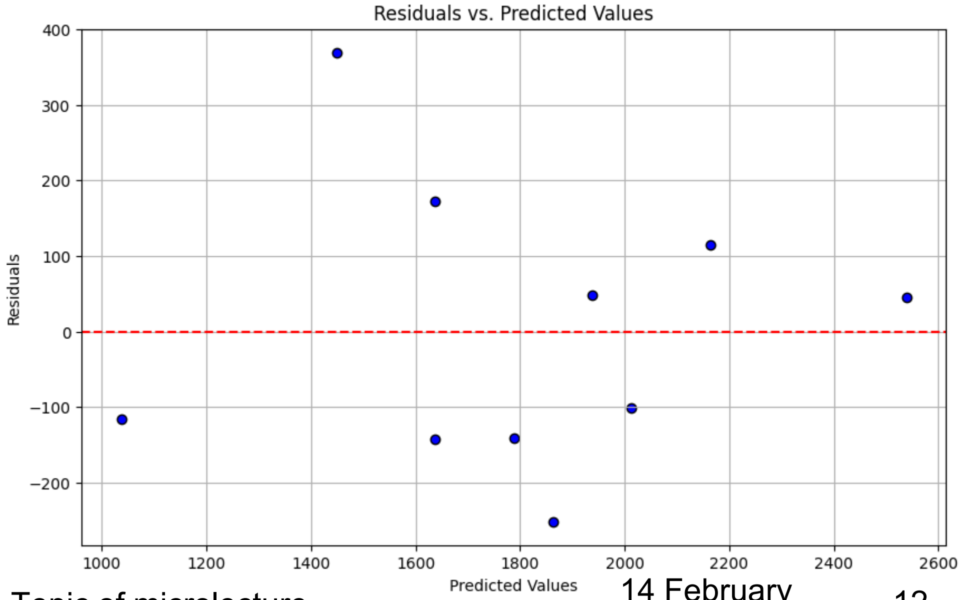
Quantity sold = f(advertising costs)



Residuum







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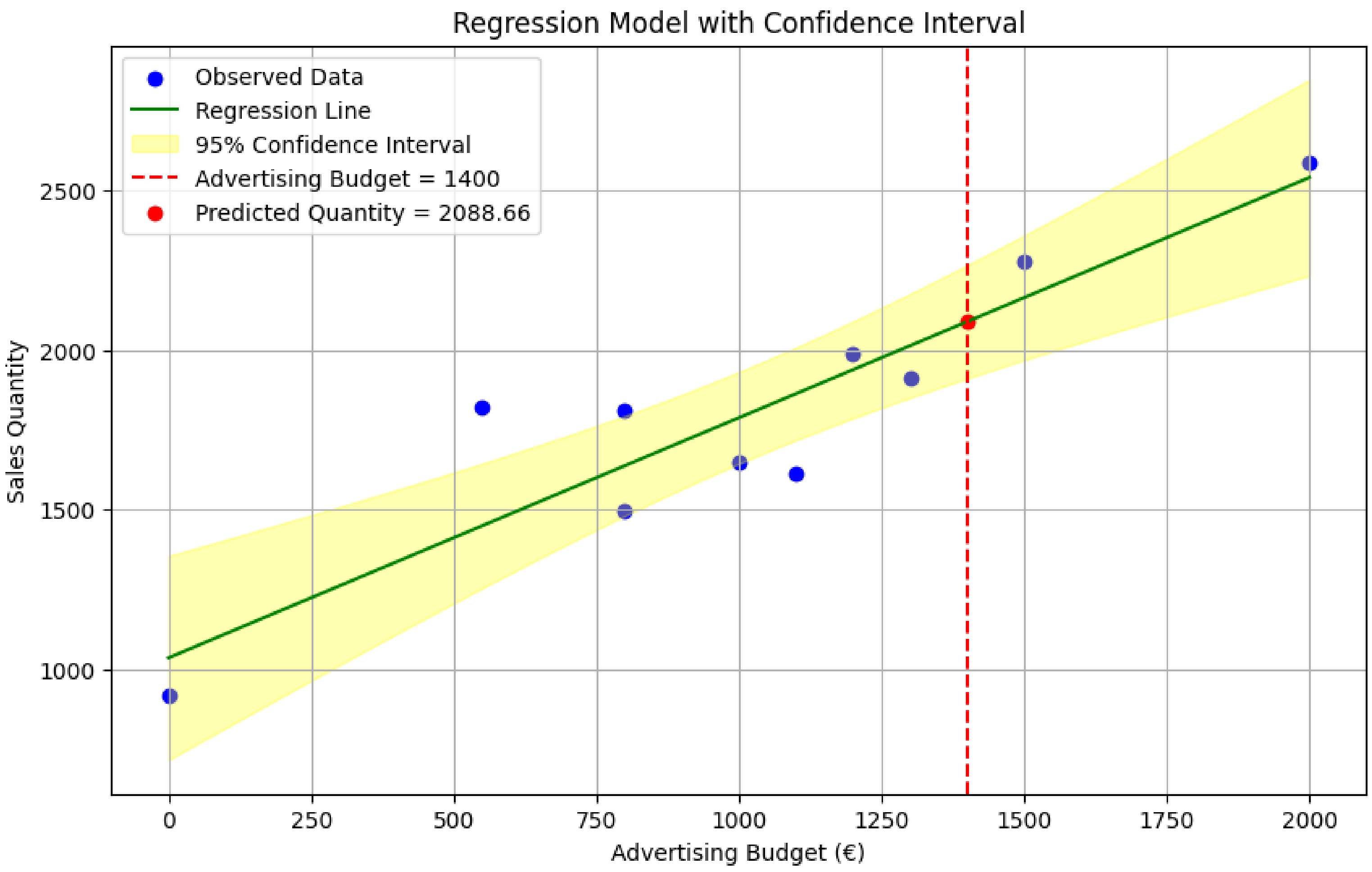
Forecast of Y values

- What value the response will have with a particular set of influence factors
- The marketing department is given a margarine advertising budget of 1,400 per month, what will be the expected turnover?

Best practice: Use interval estimates to predict the expected range of turnover.



Thank you very much for your attention!

Literature

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